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ABSTRACTS

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LIST OF TALKS

Marcin Anholcer

Poznań University of Economics and Business

Nowhere-zero group irregular labelings of graphs

We investigate the group irregularity strength, $s_g(G)$, of a graph, i.e. the least integer k such that taking any Abelian group $\mathcal{G}$ of order k, there exists a function $f : E(G) \to \mathcal{G}$ so that the sums of edge labels incident with every vertex are distinct. So far the best upper bound on $s_g(G)$ for a general graph G was exponential in $n - c$, where n is the order of G and c denotes the number of its components. In this note we prove that $s_g(G)$ is linear in n, namely not greater than $2n$. In fact, we prove a stronger result, as we additionally forbid the identity element of a group to be an edge label or the sum of labels around a vertex. We consider also locally irregular labelings where we require only sums of adjacent vertices to be distinct. For the corresponding graph invariant we prove the general upper bound: $\Delta(G) + \mathrm{col}(G) - 1$ (where $\mathrm{col}(G)$ is the coloring number of G) in the case when we do not use the identity element as an edge label, and a slightly worse one if we additionally forbid it as the sum of labels around a vertex. In the both cases we also provide a sharp upper bound for trees and a constant upper bound for the family of planar graphs.

This is joint work with Sylwia Cichacz and Jakub Przybyło.

Sylwia Antoniuk
Adam Mickiewicz University

Powers of Hamiltonian cycles in randomly augmented graphs

We investigate the existence of powers of Hamiltonian cycles in graphs with large minimum degree to which some additional edges have been added in a random manner. It follows from the theorems of Dirac and of Komlós, Sarközy, and Szemerédi, who confirmed the Pósa–Seymour conjecture, that for every $k \geq 1$ and sufficiently large n already the minimum degree $\delta(G) \geq \frac{k}{k+1}n$ for an n-vertex graph G alone suffices to ensure the existence of the k-th power of a Hamiltonian cycle. Here we show that under essentially the same degree assumption the addition of just $O(n)$ random edges ensures the presence of the $(2k+1)$-st power of a Hamiltonian cycle with probability approaching one as $n \to \infty$.

This is joint work with Andrzej Dudek, Christian Reiher, Andrzej Ruciński and Mathias Schacht.

Halina Bielak
Maria Curie Skłodowska University

Multicolour Ramsey numbers for some selected graphs

The k-colour Ramsey number $R(G_1, G_2, \ldots, G_k)$ is the smallest integer n such that in arbitrary edge k-colouring of K_n a subgraph G_i in the colour i, $1 \leq i \leq k$ is contained. The Turán number $\mathrm{ex}(n, G)$ is the maximum number of edges of a graph on n vertices which does not contain G as a subgraph. For a given $k \geq 1$ we show lower and upper bounds for $R(G_1, G_2, \ldots, G_k, C_m)$, where G_i $(1 \leq i \leq k)$ is a linear forest with small components and C_m is a cycle of order m, where $m \geq 3$. We apply Turán numbers [3-4] for counting upper bounds.

References

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Mindaugas Bloznelis

Vilnius University

Normal approximation to the number of small cliques in sparse random intersection graphs

Random intersection graphs with bounded average degree have many small cliques. I will present recent results on the normal approximation of the number of cliques of given size. An important application is the asymptotic normality of the global clustering coefficient.

This is joint work with Jerzy Jaworski.

Béla Bollobás
University of Cambridge and University of Memphis

Sieves, random Xorsat and the sharp threshold for making squares

Motivated by the fastest factoring algorithms, in 1994 Pomerance posed the problem of determining the threshold of the event that a random sequence of N integers, each chosen uniformly from the set $\{1, \ldots, x\}$, contains a subsequence, the product of whose elements is a perfect square. In 1996, Pomerance gave good bounds on this threshold and also conjectured that it is sharp.

In 2012, Croot, Granville, Pemantle and Tetali significantly improved the bounds given by Pomerance, and conjectured that their upper bound is in fact the sharp threshold. In recent work, Paul Balister, Rob Morris and I have proved this conjecture. In my talk, I shall give a brief overview of some related results and of some of the ideas used in the proof, which relies on techniques from number theory, combinatorics and stochastic processes.

Padraig Condon
University of Birmingham

A BANDWIDTH THEOREM FOR APPROXIMATE DECOMPOSITIONS

We provide a degree condition on a regular n-vertex graph G which ensures the existence of a near optimal packing of any family H of bounded degree n-vertex k-chromatic separable graphs into G. In general, this degree condition is best possible. Here a graph is separable if it has a sublinear separator whose removal results in a set of components of sublinear size. Equivalently, the separability condition can be replaced by that of having small bandwidth. Thus our result can be viewed as a version of the bandwidth theorem of Böttcher, Schacht and Taraz in the setting of approximate decompositions. In particular, this yields an approximate version of the tree packing conjecture in the setting of regular host graphs G of high degree. Similarly, our result implies approximate versions of the Oberwolfach problem, the Alspach problem and the existence of resolvable designs in the setting of regular host graphs of high degree.

This is joint work with Daniela Kühn, Jaehoon Kim and Deryk Osthus

Matthew Coulson
University of Birmingham

Rainbow Hamilton cycles in Dirac graphs

A famous theorem of Dirac states that any graph on n vertices with minimum degree at least $n/2$ has a Hamilton cycle. Such graphs are called Dirac graphs. Strengthening this result, we show the existence of rainbow Hamilton cycles in μn-bounded colourings of Dirac graphs for sufficiently small $\mu > 0$.

This is joint work with Guillem Perarnau.

Sebastian Czerwiński

University of Zielona Góra

Generalized arboricity of graphs with large girth

The *arboricity* of a graph G is the least number of colors needed to color the edges of G so that no cycle is monochromatic. We consider a higher order analog of this parameter, denoted by $\mathrm{Arb}_p(G)$, introduced recently by Nešetřil, Ossona de Mendez, and Zhu. It is defined as the least number of colors needed to color the edges of a graph so that each cycle C gets at least $\min\{|C|, p+1\}$ colors. So, $\mathrm{Arb}_1(G)$ is the usual arboricity of G, while $\mathrm{Arb}_2(G)$ can be seen as a relaxed version of the *acyclic chromatic index* of G. By the results of Nešetřil, Ossona de Mendez, and Zhu it follows that $\mathrm{Arb}_p(G)$ is bounded for classes of graphs with *bounded expansion*, provided that the girth of graphs G is sufficiently large (depending on p). Using more direct approach we obtain explicit upper bounds on $\mathrm{Arb}_p(G)$ for some classes of graphs. In particular, we prove that $\mathrm{Arb}_p(G) \leq p+1$ for every planar graph of girth at least 2^{p+1}. A similar result holds for graphs with arbitrary fixed genus. We also demonstrate that $\mathrm{Arb}_2(G) \leq 5$ for outerplanar graphs, which is best possible. By using *entropy compression* argument we prove that $\mathrm{Arb}_p(G) \leq (\Delta-1)p+1$ for graphs of maximum degree Δ and sufficiently large girth.

Dariusz Dereniowski
Gdańsk University of Technology

On-line tree exploration strategies

We discuss an on-line tree exploration problem in which, in graph-theoretic terms, the goal is to cover an input tree with a collection of paths, each having bounded length. It is required that each path starts at a specified vertex of the tree, the root, and depending on the variant of the problem, each path either must also end at the root or may end at an arbitrary vertex. In this talk we survey some recent results and discuss some open problems.

Alberto Espuny Díaz

University of Birmingham

Edge correlations in random regular hypergraphs

Compared to the classical binomial random (hyper)graph model, the study of random regular hypergraphs is made more challenging due to correlations between the occurrence of different edges. We develop an edge-switching technique for hypergraphs which allows us to show that these correlations are limited for a large range of densities. This extends some previous results of Kim, Sudakov and Vu for graphs. From our results we deduce several corollaries on subgraph counts in random d-regular hypergraphs. We also prove a conjecture of Dudek, Frieze, Ruciński and Šileikis on the threshold for the existence of an ℓ-overlapping Hamilton cycle in a random d-regular r-graph. As an application, we use our counting results to prove bounds on the query complexity of testing subgraph-freeness.

This is joint work with Felix Joos, Daniela Kühn and Deryk Osthus.

Hanna Furmańczyk
University of Gdańsk

Equitable colorings of hypergraphs

A graph is equitably k-colorable if its vertices can be partitioned into k sets/color classes in such a way that monochromatic edges are avoided and the number of vertices in any two color classes differs by at most one. In the talk we generalize this definition for hypergraphs. We prove that the problem of equitable 2-coloring of hypergraphs is NP-complete even for 3-uniform hyperstars. Finally, we apply the method of dynamic programming for designing polynomial-time algorithm to equitably k-color linear hypertrees, where $k \geq 2$ is fixed.

Przemysław Gordinowicz

Łódź University of Technology

Some properties of automorphism group of countable homogeneous posets

A countable relational structure is homogeneous if every isomorphism between its finite substructures can be extended to an automorphism. If A is such a structure, then the group $\mathrm{Aut}(A)$ has a natural topology (basis consists of sets of possible extensions of each isomorphism between finite substructures) in which $\mathrm{Aut}(A)$ is a Polish topological group. We study properties of automorphism group of countably infinite homogeneous posets. In particular we show that each such a group contains a dense free subgroup of two generators. In our considerations we use the result of Schmerl which says that there are essentially four types of countable homogeneous posets.

This is joint work with Marek Bienias, Szymon Głąb and Filip Strobin.

Izolda Gorgol
Lublin University of Technology

Induced Ramsey numbers for multiple copies of graphs

We say that a graph F strongly arrows a pair of graphs (G, H) if any colouring of its edges with red and blue leads to either a red G or a blue H appearing as induced subgraphs of F. The induced Ramsey number, $\mathrm{IR}(G, H)$ is defined as the smallest order of a graph that strongly arrows (G, H). We consider the connection between the induced Ramsey number for a pair of two connected graphs $\mathrm{IR}(G, H)$ and the induced Ramsey number for multiple copies of these graphs $\mathrm{IR}(sG, tH)$, where xG denotes the pairwise vertex-disjoint union of x copies of G. It is easy to see that $\mathrm{IR}(sG, tH)$ is at most $(s+t-1)\,\mathrm{IR}(G, H)$. For all known results on induced Ramsey numbers for multiple copies, the inequality above holds as equality. We show that there are infinite classes of graphs for which the inequality above is strict. On the other hand, we provide further examples of classes of graphs for which the inequality above holds as equality.

This is joint work with Maria Axenovich.

Mariusz Grech
University of Wrocław

Decompositions of the automorphism groups of edge-colored graphs into the direct product of permutation groups

In the paper 'Graphical complexity of products of permutation groups', M. Grech, A. Jeż, and A. Kisielewicz have proved that the direct product of automorphism groups of edge-colored graphs is itself the automorphism groups of an edge-colored graph. Now, we study the direct product of two permutation groups such that at least one of them fails to be the automorphism group of an edge-colored graph. We find necessary and sufficient conditions for the direct product to be the automorphism group of an edge-colored graph. The same problem is solved for the edge-colored digraphs.

Jarosław Grytczuk
Warsaw University of Technology

On some coloring problems of the integers

I will present some of my favorite problems on coloring the integers. In most of them a central role is played by *central arithmetic progressions*, that is, sets of the form $\{a, 2a, \ldots, ka\}$ consisting of initial multiples of a fixed natural number. Is it true, for instance, that for every k, there is a k-coloring of positive integers such that every central arithmetic progression of length k is rainbow? As one may expect, problems of this type inevitably lead to deep number-theoretic issues. I will formulate some new questions connected to Erdős' Discrepancy Problem and Riemann Hypothesis.

Andrzej Grzesik

University of Warwick and Jagiellonian University

Forcing cycles in directed graphs

Motivated by the well-known Caccetta-Häggkvist Conjecture, Kelly, Kühn and Osthus made a conjecture on minimal semidegree forcing appearance of a directed cycle of a given length and proved it for cycles of length not divisible by 3. In the talk we will present an overview of a proof of all the remaining cases of their conjecture.

This is joint work with Roman Glebov and Jan Volec.

Grzegorz Gutowski
Jagiellonian University

Acyclic edge-coloring using entropy compression

Let G be any graph with maximum degree d. We show a randomized procedure that colors edges of G so that:

1/ two adjacent edges get two different colors;

2/ edges of any cycle get at least three different colors.

Such a coloring is called an acyclic edge-coloring of G. The minimum number of colors required for such a coloring is called the acyclic index of G. It is conjectured that acyclic index of G is at most $d + 2$. Esperet and Parreau used entropy compression method to show that acyclic index of G is at most $4d - 4$. We use similar methods to present a coloring procedure that achieves better multiplicative factor.

This is joint work with Jakub Kozik and Xuding Zhu.

Tony Johansson
Uppsala University

The cover time of a biased random walk on the random regular graph

On a finite graph G, we consider a random walk which prefers to use previously unvisited edges whenever possible. Suppose all edges are initially coloured red. At any stage of the process, the walk moves from its current vertex along a red edge chosen uniformly at random, recolouring it blue. If there are no red edges to traverse, then it walks along a blue edge chosen uniformly at random. We show that if G is chosen uniformly at random from the set of simple r-regular graphs for some fixed odd r, then the expected vertex cover time of this walk is asymptotically $n\ln(n)/(r-2)$.

Pawaton Kaemawichanurat

King Mongkut's University of Technology Thonburi

Connected domination critical graphs with cut vertices

A graph G is said to be k connected domination critical if the connected domination number of G is k but the connected domination number of $G+uv$ is less than k for any pair of non-adjacent vertices u and v of G. Let G be a k connected domination critical graph with $a(G)$ cut vertices. It was proved that, for $k = 3$ or 4, every k connected domination critical graph has $a(G)$ at most $k - 2$. In this talk, we generalize that every k connected domination critical graph has $a(G)$ at most $k - 2$ for all k greater than 4. We also characterize all k connected domination critical graphs when $a(G)$ is achieving the upper bound.

Nina Kamčev
Swiss Federal Institute of Technology in Zürich

Intervals in the Hales-Jewett theorem

The Hales-Jewett Theorem states that any r-colouring of $[m]^n$ contains a monochromatic combinatorial line if n is sufficiently large. Shelah's proof of the theorem for $m = 3$ yields a monochromatic combinatorial line in $[3]^n$ whose set of wildcard coordinates is the union of *at most* r intervals. The question of how optimal this structure is has been investigated by Conlon, Kamčev, Leader, Räty and Spiegel, revealing a surprising alternation depending on the parity of r. Namely, for odd r, there are colourings avoiding all monochromatic lines whose wildcard set consists of fewer than r intervals. However, for even r and large n, we can achieve $r - 1$ intervals.

In this talk, we will outline some of the ideas involved in the proofs, focusing on the upper bound for even r. We hope that our approach could also be relevant to the corresponding problem for larger alphabets ($m \geq 4$).

Bartłomiej Kielak
Polish Academy of Sciences

On the maximum number of odd cycles in graphs without smaller odd cycles

We prove that for each odd integer $k \geq 7$, every graph on n vertices without odd cycles of length less than k contains at most $(n/k)^k$ cycles of length k. This generalizes the previous results on the maximum number of pentagons in triangle-free graphs, conjectured by Erdős in 1984, and asymptotically determines the generalized Turán number $\mathrm{ex}(n, C_k, C_{k-2})$ for odd k. In contrary to the previous results on the pentagon case, our proof is not computer-assisted.

This is joint work with Andrzej Grzesik.

Andrzej Kisielewicz
University of Wrocław

König's problem on permutation groups

Jakub Konieczny

The Hebrew University of Jerusalem and Jagiellonian University

Digital sequences from the perspective of additive combinatorics

Automatic sequences are among the most basic models of computation. Intuitively speaking, an automatic sequence is one whose n-th term can be computed from the digits of n using a finite device. Perhaps the simplest example of a (non-trivial) automatic sequence carries the name of Thue–Morse and is given by $t(n) = (-1)^{s_2(n)}$, where $s_2(n)$ denotes the sum of digits of n base 2. It is also a 2-multiplicative sequence, meaning that $t(n)$ can be computed as the product of contributions which depend on consecutive binary digits of n (each 1 in the expansion contributes the factor of -1). The talk will concern these and other classes of sequences defined in terms of the digits of the input.

Various uniformity properties of such sequences have long been studied. It was already in 1968 that Gelfond obtained quantitative estimates of the Fourier coefficients of the Thue–Morse sequence, implying in particular that in any sufficiently long arithmetic progression the sum of binary digits is even for approximately half of the terms. A more comprehensive inquiry was later undertaken by Mauduit and Sarközy, and behaviour of more general sequences restricted to more general index sets (such as the primes or the values of integer polynomials) has been extensively studied by many authors, including Drmota, Mauduit, Müllner, Rivat, Spiegelhofer and others.

With the advent of higher order Fourier analysis, new notions of uniformity have come to light. Specifically, a sequence f can be construed as uniform (or pseudorandom) of order $s \geq 2$ if the Gowers uniformity norms $\|f\|_{U^s[N]}$ become small as $N \to \infty$. The usefulness of this notion stems in large part from the fact that the Gowers norms control the count of arithmetic progressions (as well as other linear patterns) — indeed, this observation lies at the heart of Gower's proof of Szemerédi theorem and the later work of Green and Tao on linear patterns in the primes.

During the talk I will discuss some new and old results concerning combinatorial properties of various sequences defined in terms of digital expansions. In particular, we will be concerned with the conditions under which such sequences are Gowers uniform.

This includes joint work with Jakub Byszewski, Aihua Fan and Tanja Eisner.

Alexandr V. Kostochka

University of Illinois at Urbana-Champaign

Extremal problems on paths and matchings in ordered and convex geometric hypergraphs

An *ordered r-graph* is an r-uniform hypergraph whose vertex set is linearly ordered, and *a convex geometric r-graph* (cg r -graph, for short) is an r-uniform hypergraph whose vertex set is cyclically ordered. Extremal problems for ordered and cg graphs have a rich history.

We consider similar extremal problems for two types of paths and matchings in ordered r-graphs and cg r-graphs: *zigzag* and *crossing* paths and matchings. We prove bounds on Turán numbers for these configurations; some of them are exact. Our theorem on zigzag paths in cg r-graphs is a common generalization of early results of Hopf and Pannwitz, Sutherland, Kupitz and Perles for cg graphs. It also yields the current best bound for the extremal problem for tight paths in uniform hypergraphs. There are interesting similarities and differences between the ordered setting and the convex geometric setting.

This is joint work with Zoltan Füredi, Tao Jiang, Dhruv Mubayi and Jacques Verstraëte.

Patryk Kozieł
Wrocław University of Science and Technology

Uniform random posets

We propose a simple algorithm generating labelled posets of given size according to the almost uniform distribution. By "almost uniform" we understand that the distribution of generated posets converges in total variation to the uniform distribution. Our method is based on a Markov chain generating directed acyclic graphs.

Jakub Kozik
Jagiellonian University

Coloring algorithms for small hypergraphs

Mariusz Kwiatkowski
University of Warmia and Mazury

The graphs of projective codes

Consider the Grassmann graph formed by k-dimensional subspaces of an n-dimensional vector space over the field of q elements ($1 < k < n-1$), two subspaces are adjacent if their intersection is $k-1$ dimensional. Denote by $\Pi(n,k)_q$ the restriction of this graph to the set of projective $[n,k]_q$ codes, i.e. linear codes whose generator matrices do not contain proportional columns. In the case when $q \geq \binom{n}{2}$, we show that the graph $\Pi(n,k)_q$ is connected, its diameter is equal to the diameter of the Grassmann graph and the distance between any two vertices coincides with the distance between these vertices in the Grassmann graph. Also, we give some observations concerning the graphs of simplex codes. For example, binary simplex codes of dimension 3 are precisely maximal singular subspaces of a non-degenerate quadratic form.

Richard Lang

University of Birmingham

Monochromatic cycle partitions in random graphs

Erdős, Gyárfás and Pyber showed that every r-edge-coloured complete graph K_n can be covered by $25r^2 \log r$ vertex-disjoint monochromatic cycles (independent of n). We extend this result to the setting of binomial random graphs. That is, we show that if $p = p(n) \gg n^{-1/(2r)}$, then with high probability any r-edge-coloured $G(n,p)$ can be covered by at most $1000r^4 \log r$ vertex-disjoint monochromatic cycles.

Robert Malona
University of Opole

Hamiltonicity in tournaments with scores

The classic Rédei theorem states that every tournament has got a Hamiltonian path and the classic Camion theorem states that every strongly connected tournament has got a Hamiltonian cycle. In this lecture we will consider tournaments with draws and we will show that for some natural class of such tournaments with draws the modified versions of the two theorems hold.

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Tomáš Masařík
Charles University

Parameterized approximation schemes for Steiner trees with small number of Steiner vertices

We study the Steiner Tree problem, in which a set of terminal vertices needs to be connected in the cheapest possible way in an edge-weighted graph. This problem has been extensively studied from the viewpoint of approximation and also parametrization. In particular, on one hand Steiner Tree is known to be APX-hard, and $W[2]$-hard on the other, if parameterized by the number of non-terminals (Steiner vertices) in the optimum solution. In contrast to this we give an efficient parameterized approximation scheme (EPAS), which circumvents both hardness results. Moreover, our methods imply the existence of a polynomial size approximate kernelization scheme (PSAKS) for the considered parameter. We further study the parameterized approximability of other variants of Steiner Tree, such as Directed Steiner Tree and Steiner Forest. For neither of these an EPAS is likely to exist for the studied parameter: for Steiner Forest an easy observation shows that the problem is APX-hard, even if the input graph contains no Steiner vertices. For Directed Steiner Tree we prove that approximating within any function of the studied parameter is $W[1]$-hard. Nevertheless, we show that an EPAS exists for Unweighted Directed Steiner Tree, but a PSAKS does not. We also prove that there is an EPAS and a PSAKS for Steiner Forest if in addition to the number of Steiner vertices, the number of connected components of an optimal solution is considered to be a parameter. We also mention some applications of our algorithmic approach in practice.

Tamás Mészáros
Free University of Berlin

Boolean dimension and tree-width

The dimension is a key measure of complexity of partially ordered sets. Small dimension allows succinct encoding. Indeed if P has dimension d, then to know whether $x \leq y$ in P it is enough to check whether $x \leq y$ in each of the d linear extensions of a witnessing realizer. Focusing on the encoding aspect Nešetřil and Pudlák defined a more expressive version of dimension. A poset P has Boolean dimension at most d if it is possible to decide whether $x \leq y$ in P by looking at the relative position of x and y in only d permutations of the elements of P. We prove that posets with cover graphs of bounded tree-width have bounded Boolean dimension. This stays in contrast with the fact that there are posets with cover graphs of tree-width three and arbitrarily large dimension. This result might be a step towards a resolution of the long-standing open problem: Do planar posets have bounded Boolean dimension?

Piotr Micek

Jagiellonian University

Unavoidable minors of 2-connected graphs with large pathwidth

We prove the conjecture of Seymour (1993) that for every apex-forest H_1 and outerplanar graph H_2 there is an integer p such that every 2-connected graph of pathwidth at least p contains H_1 or H_2 as a minor. An independent proof was recently obtained by Dang and Thomas.

This is joint work with Tony Huynh, Gwenaël Joret and David R. Wood.

Patryk Mikos
Jagiellonian University

FirstFit on-line coloring of short intervals with bandwidths

The online interval coloring and its variants are important combinatorial problems with many practical applications in network multiplexing, resource allocation and job scheduling. In this talk we investigate a performance of the simplest greedy algorithm in a variant of the online interval coloring problem in which all intervals have lengths in $[1, \sigma]$ and weights in $[\alpha, \beta]$.

Mateusz Miotk

University of Gdańsk

Certified domination

A set D of vertices of a graph $G = (V_G, E_G)$ is a *dominating set* of G if every vertex in $V_G - D$ is adjacent to at least one vertex in D. The *domination number* (*upper domination number*, respectively) of a graph G, denoted by $\gamma(G)$ ($\Gamma(G)$, respectively), is the cardinality of a smallest (largest minimal, respectively) dominating set of G. A subset $D \subseteq V_G$ is called a *certified dominating set* of G if D is a dominating set of G and every vertex in D has either zero or at least two neighbors in $V_G - D$. The cardinality of a smallest (largest minimal, respectively) certified dominating set of G is called the *certified* (*upper certified*, respectively) *domination number* of G and is denoted by $\gamma_{\rm cer}(G)$ ($\Gamma_{\rm cer}(G)$, respectively). In this presentation we show basic properties of Certified Domination and we present relations between domination, upper domination, certified domination and upper certified domination numbers of a graph.

Suchisimita Mishra

Indian Institute of Technology Madras

Strong chromatic index of graph products

A strong edge coloring of a graph is an assignment of colors to edges such that vertices of every color class induce a matching. The smallest number of colors for which a strong edge coloring of a graph G exists is known as the strong chromatic index and it is denoted by $\chi'_s(G)$. In 1985, Erdős and Nešetřil conjectured that every graph G with the maximum degree Δ satisfies, $\chi'_s(G) \leq \frac{5}{4}\Delta^2$ if Δ is even and $\chi'_s(G) \leq (5\Delta^2 - 2\Delta + 1)/4$ if Δ is odd. In this work we obtain upper and lower bounds on the Cartesian products of trees and tree with cycle.

Piotr Miska

Jagiellonian University

p-ADIC DENSENESS OF MEMBERS OF PARTITIONS OF $\mathbb{N}$ AND p-ADIC ASYMPTOTIC DENSITIES

The *ratio set* of a set of positive integers A is defined as $R(A) := \{a/b : a, b \in A\}$. The study of the denseness of $R(A)$ in the set of positive real numbers is a classical topic and, more recently, the denseness in the set of p-adic numbers $\mathbb{Q}_p$ has also been investigated. Let $A_1, \dots, A_k$ be a partition of $\mathbb{N}$ into k sets. We will show that for all prime numbers p but at most $\lfloor \log_2 k \rfloor$ exceptions at least one of $R(A_1), \dots, R(A_k)$ is dense in $\mathbb{Q}_p$. Moreover, we will present the result that for all prime numbers p but at most $k-1$ exceptions at least one of $A_1, \dots, A_k$ is dense in $\mathbb{Z}_p$. Both these results are optimal in the sense that there exist partitions $A_1, \dots, A_k$ having exactly $\lfloor \log_2 k \rfloor$, respectively $k-1$, exceptional prime numbers; and we give explicit constructions for them. The above results imply that in any finite partition of the set of positive integers there is at least one subset dense in $\mathbb{Z}_p$ (or with dense ratio set in $\mathbb{Q}_p$, respectively) for all but finitely many prime numbers p. It resembles theorems from Ramsey theory that each finite partition of the set of positive integers contains at least one subset which has rich combinatorial structure, in appropriate sense. In particular, they are similar to famous van der Waerden's theorem that in every finite partition of positive integers there exists a member of this partition containing arbitrarily long finite arithmetic progressions (we will call such a set as AP-rich, for short). Van der Waerden's theorem is easily implied by Szemerédi's theorem, as each finite partition of positive integers has at least one set with positive upper asymptotic density. We thus see that in case of van der Waerden's theorem we have some subadditive measure on $\mathbb{N}$ that its positive value ensures AP-richness of a given set. Existence of such measure is a motivation for us to find a subadditive measures that ensures denseness of a given set in $\mathbb{Z}_p$ (or denseness of its ratio set in $\mathbb{Q}_p$, respectively), where p is a fixed prime number. During the talk we will introduce p-adic asymptotic densities and present some their properties.

The first part of the talk is joint work with Carlo Sanna.

Adam Morawiec
University of Wrocław

On the structure of Steinhaus boards

Recently, I have introduced Steinhaus boards as a combinatorial tool to solve an old, seemingly elementary problem about rooks on the chess-board, posed in the 1950s by Hugo Steinhaus in Matematyka, Polish mathematical journal for schools. This problem appeared to have deep and multiple connections with one of the most famous theorems in combinatorics, viz. Hall's marriage theorem. I have already presented, discussed and analyzed those connections in a series of papers elsewhere. In that discussion, I have indicated that there seems to have been two last problems left open with respect to Steinhaus boards: the description of their structure and the characterization of minimal Steinhaus boards. In this paper, I would like to present a complete solution to the first of those problems, ie. to give a complete description of the structure of Steinhaus boards.

Dhruv Mubayi
University of Illinois at Chicago

New developments in hypergraph Ramsey theory

I will discuss recent progress on a variety of old and new problems in hypergraph Ramsey theory. These include classical Ramsey numbers, and various questions of Erdős, Hajnal, Gyárfás and Rogers dating back to the 1970s. Most of the new results I will describe involve constructions that include a combination of random and deterministic methods.

Carole Muller

Université Libre de Bruxelles

Excluded minors for embeddings of metric graphs in ℓ_∞^k-spaces

By *metric graph* we mean a pair (G, d), where $G = (V, E)$ is a graph and $d : E \to \mathbb{R}_{\geq 0}$ is a distance function on E. An *isometric embedding* of the metric graph (G, d) in $\ell_\infty^k = (\mathbb{R}^k, d_\infty)$ is a set of vectors $\{q_v\}_{v \in V} \subseteq \mathbb{R}^k$ such that $d_\infty(q_v, q_w) = d(vw)$ for all $vw \in E$. The graph parameter $f_\infty(G)$ is the least integer k such that there exists an isometric embedding of (G, d) in ℓ_∞^k for all distance functions d. The property $f_\infty(G) \leq k$ is closed under taking minors. By the Graph Minor Theorem of Robertson and Seymour, there exists a finite list of excluded minors for the property $f_\infty(G) \leq k$. We present four graph families of unbounded f_∞-value and show that they are unavoidable in *any* graph with large f_∞-value. This can be viewed as an approximate characterization of graphs with large f_∞-value in the same vein as the grid theorem for graphs of large treewidth.

Karolina Okrasa
Warsaw University of Technology

Intersecting edge distinguishing colorings of hypergraphs

An edge labeling of a graph distinguishes neighbors by sets (multisets, resp.), if for any two adjacent vertices u and v the sets (multisets, resp.) of labels appearing on edges incident to u and v are different. In an analogous way we define total labelings distinguishing neighbors by sets or multisets: for each vertex, we consider labels on incident edges and the label of the vertex itself. We show that these problems, and also other problems of similar flavor, admit an elegant and natural generalization as a hypergraph coloring problem. An ieds-coloring (iedm-coloring, resp.) of a hypergraph is a vertex coloring, in which the sets (multisets, resp.) of colors, that appear on every pair of intersecting edges are different. We show upper bounds on the size of lists, which guarantee the existence of an ieds- or iedm-coloring, respecting these lists. The proof is essentially a randomized algorithm, whose expected time complexity is polynomial. As corollaries, we derive new results concerning the list variants of graph labeling problems, distinguishing neighbors by sets or multisets. We also show that our method is robust and can be easily extended for different, related problems.

Mark Pankov
University of Warmia and Mazury

Z-knotted triangulations

An embedded graph is z-knotted if it has a single zigzag (up to reverse). Our main result: every triangulation has a z-knotted shredding. The proof is based on the new concept of z-monodromy.

Marcin Pilipczuk
University of Warsaw

The square root phenomenon: subexponential algorithms in sparse graph classes

Joanna Polcyn
Adam Mickiewicz University

The structure of large hypergraphs with forbidden intersections

We show that for infinitely many natural numbers k there are k-uniform hypergraphs which admit a 'rescaling phenomenon'. To be more precise, let $\mathcal{A}(k, I, n)$ denote the class of k-graphs on n vertices in which the sizes of all pairwise intersections of edges belong to a set I. We show that if $k = rt^2$ for some $r \geq 1$ and $t \geq 2$, and I is chosen in some special way, the densest graphs in $\mathcal{A}(rt^2, I, n)$ are either dominated by stars of large degree, or basically, they are 't-thick' rt^2-graphs in which vertices are partitioned into groups of t vertices each and every edge is a union of tr such groups. It is easy to see that, unlike in stars, the maximum degree of t-thick graphs is of a lower order than the number of its edges. Thus, if we study the graphs from $\mathcal{A}(rt^2, I, n)$ with a prescribed number of edges m which minimise the maximum degree, around the value of m which is the number of edges of the largest t-thick graph, a rapid, discontinuous phase transition can be observed. Interestingly, these two types of k-graphs determine the structure of all hypergraphs in $\mathcal{A}(rt^2, I, n)$. Namely, we show that each such hypergraph can be decomposed into a t-thick graph H_T, a special collection H_S of stars, and a sparse 'left-over' graph H_R.

This is joint work with Tomasz Łuczak and Christian Reiher.

Paweł Prałat
Ryerson University

k-REGULAR SUBGRAPHS NEAR THE k-CORE THRESHOLD OF A RANDOM GRAPH

We prove that the binomial random graph $G_{n,p=c/n}$ with high probability has a k-regular subgraph if c is at least $e^{-\Theta(k)}$ above the threshold for the appearance of a subgraph with minimum degree at least k; i.e. an non-empty k-core. In particular, this pins down the threshold for the appearance of a k-regular subgraph to a window of size $e^{-\Theta(k)}$.

This is joint work with Dieter Mitsche and Mike Molloy.

Michał Przykucki

University of Birmingham

Shotgun assembly of the hypercube

In recent work, Mossel and Ross, and others have considered the shotgun assembly problem in various settings. We consider shotgun assembly of the hypercube - given the r-balls of a random q-colouring of the vertices, can we reconstruct the colouring up to an automorphism with high probability? We show that for 2-balls, $q = 2$ is sufficient, and that for 1-balls, $q \geq n^{2+\Theta(\log^{-\frac{1}{2}} n)}$ is sufficient. Along the way we also prove some stability results for set systems.

This is joint work with Alexander Roberts and Alex Scott.

Paweł Rzążewski

Warsaw University of Technology

Fine-grained complexity in geometric intersection graphs

A typical problem in the classical complexity theory is to characterize, which members of a given family of computational problems can be solved in polynomial time, and which of them are NP-hard. Problems classified as NP-hard are often considered unsolvable in practice and thus not interesting.

In past two decades, algorithmic community started paying much more attention to problems, which are computationally hard. Many new algorithmic tools were developed and allowed to design algorithms that are significantly faster than a trivial brute-force approach. However, some well-known problems resist such algorithms. For example, we still cannot find a subexponential algorithm for 3-Sat, i.e., an algorithm solving any instance of 3-Sat with n variables and m clauses in time $(n+m)^{\mathcal{O}(1)} \cdot 2^{o(n+m)}$. The *Exponential Time Hypothesis* by Impagliazzo and Paturi (1999) asserts that such an algorithm cannot exist.

The ETH became one of standard assumptions in complexity theory. Being stronger than "P $\neq$ NP", the ETH allows for a much finer classification of problems that are NP-hard. We can distinguish the class of problems that are "easier", i.e., admit subexponential algorithms, and "harder", i.e., cannot be solved in subexponential time. We can also ask for complexity of an optimal algorithm and try to find matching upper (algorithmic) and lower (complexity) bounds.

In this talk, we will discuss how deep structural analysis of intersection graphs of geometric objects may lead to tight or almost tight algorithmic results. In particular, we will discuss subexponential algorithms for coloring intersection graphs of disks and, in general d-dimensional balls, or finding maximum cliques in intersection graphs of disks. We will also show some natural problems that can be solved in subexponential time in intersection graphs of segments, and some which do not admit such algorithms.

Wojciech Samotij
Tel Aviv University

Entropy and counting

Nicolás Sanhueza Matamala
University of Birmingham

An asymptotic bound for the strong chromatic number

The strong chromatic number $\chi_s(G)$ of a graph G on n vertices is the least number r with the following property: after adding $r\lceil n/r\rceil - n$ isolated vertices to G and taking the union with any collection of spanning disjoint copies of K_r in the same vertex set, the resulting graph has a proper vertex-colouring with r colours. We show that for every $c > 0$ and every graph G on n vertices with $\Delta(G) > cn$, $\chi_s(G) \leq (2+o(1))\Delta(G)$, which is asymptotically best possible.

Oriol Serra

BarcelonaTech

Extremal additive sets

By the well-known Freiman-Ruzsa theorem, a set A of integers with small doubling, $|A+A| < c|A|$, is a dense subset of a multidimensional arithmetic progression P. The volume of A is the smallest volume of such a progression containing A.

Improving on earlier results by Freiman, Bilu, Chang, Sanders and Konyagin, Schoen obtained an upper bound on the volume of A in terms of the doubling constant c which is asymptotically best possible.

In this context, a set of integers is said to be additively extremal if it has the largest volume among all sets with the same cardinality and doubling. Freiman conjectured an exact formula for this maximum volume. The talk will survey results on this conjecture and on the structure of extremal additive sets.

This is joint work with Gregory A. Freiman.

Michał Seweryn
Jagiellonian University

The dimension of posets with excluded minors in cover graphs

A fan is a graph obtained from a path by adding an extra vertex adjacent to all vertices on the path. We give a qualitative structure theorem for graphs excluding a fan as a minor. This is inspired by a recent result by Ding that gives an approximate description of graphs excluding $K_{2,n}$ as a minor. Next, we use both characterizations to show that the dimension of a poset is bounded in terms of the size of a largest $K_{2,n}$ or a fan (graph) which is a minor of the cover graph. This is a step towards characterization of minor-closed graph classes such that posets with cover graphs from such a class has bounded dimension.

Shahriar Shahriari

Pomona College

Forbidden configurations for posets of subspaces

Let V be a finite dimensional vector space over a finite field, and consider the poset of subspaces of V ordered by inclusion. The combinatorial properties of this partially ordered set often resemble those of the Boolean lattices, the subsets of a finite set ordered by inclusion. Often the case of subsets is easier to handle, but, there are situations where a combinatorial question is easier to answer for subspaces. In this talk, we discuss a few such problems including forbidden configuration problems: Given a small poset P, what is the largest number of subspaces of V that do not contain a copy of P?

Fiona Skerman

Uppsala University

Permutations in binary trees and split trees

We investigate the number of permutations that occur in random node labellings of trees. This is a generalisation of the number of subpermutations occuring in a random permutation. It also generalises some recent results on the number of inversions in randomly labelled trees [1]. We consider complete binary trees as well as random split trees a large class of random trees of logarithmic height introduced by Devroye [2]. Split trees consist of nodes (bags) which can contain balls and are generated by a random trickle down process of balls through the nodes.

In the case of the complete binary trees that asymptotically the cumulants of the number of occurrences of a fixed permutation in the random node labelling have explicit formulas. For a random split tree with high probability we show the cumulants of the number of occurrences are asymptotically an explicit parameter of the split tree. I will describe some results on the number of embeddings of digraphs into split trees, used in the proof of the second result, which may be of independent interest.

This is joint work with Michael Albert, Cecilia Holmgren, Tony Johansson.

References

[1] X. Cai, C. Holmgren, S. Janson, T. Johansson and F. Skerman, *Inversions in split trees and conditional Galton–Watson trees*, arXiv preprint arXiv:1709.00216 2017.

[2] L. Devroye, *Universal limit laws for depths in random trees*, SIAM Journal on Computing, 28(2), 1998, 409-432.

Joanna Sokół
Warsaw University of Technology

Online coloring of intersection graphs of various shapes

We study intersection graphs of geometric shapes such as discs or rectangles in $\mathbb{R}^2$, intervals in $\mathbb{R}$ etc. We allow shapes to be translated, rotated and even scaled, but not too much. Say the diameters of copies of a chosen shape is between 1 and σ, where $\sigma \geq 1$. What can we say about online coloring of an intersection graph of such shapes? We try to establish bounds on the competitive ratio of online coloring algorithms for these graphs in terms of σ.

Miloš Stojaković
University of Novi Sad

Semi-random graph process

We introduce and study a novel semi-random multigraph process, described as follows. The process starts with an empty graph on n vertices. In every round of the process, one vertex v of the graph is picked uniformly at random and independently of all previous rounds. We then choose an additional vertex (according to a strategy of our choice) and connect it by an edge to v. For various natural monotone increasing graph properties P, we give tight upper and lower bounds on the minimum (extended over the set of all possible strategies) number of rounds required by the process to obtain, with high probability, a graph that satisfies P. Along the way, we show that the process is general enough to approximate (using suitable strategies) several well-studied random graph models.

This is joint work with Omri Ben-Eliezer, Dan Hefetz, Gal Kronenberg, Olaf Parczyk and Clara Shikhelman

Konstanty Junosza-Szaniawski
Warsaw University of Technology

Online coloring of short intervals

We study the online graph coloring problem restricted to the intersection graphs of intervals with lengths in $[1;\sigma]$. For $\sigma = 1$ it is the class of unit interval graphs, and for $\sigma = \infty$ the class of all interval graphs. Our focus is on intermediary classes. We present a $(1+\sigma)$-competitive algorithm, which beats the state of the art for $1 < \sigma < 2$. For $\sigma = 1$ our algorithm matches the performance of FirstFit, which is 2-competitive for unit interval graphs. For $\sigma = 2$ our algorithm matches the Kierstead-Trotter algorithm, which is 3-competitive for all interval graphs. On the lower bound side, we prove that no algorithm is better than 5/3-competitive for any $\sigma > 1$, nor better than 7/4-competitive for any $\sigma > 2$, and that no algorithm beats the 5/2 asymptotic competitive ratio for all, arbitrarily large, values of σ.

Jacinta Torres
Karlsruhe Institute of Technology

Branching rules and bijections

A branching rule describes the algebraic behaviour of an irreducible representation restricted to the action of a smaller group. This behaviour is often understood in a combinatorial way. We present a branching rule obtained in joint work with Bea Schumann, provide a promenade through the bijections that compose its proof, and shine light on work to be done. A lot of tableaux combinatorics are involved.

Tom Trotter

Georgia Institute of Technology

Two challenging conjectures for planar posets

We discuss recent progress towards the resolution of two long standing problems involving planar posets. The first is due to Nešetřil and Pudlák who conjectured that the Boolean dimension is bounded on the class of planar posets. Most likely I am responsible for the second problem which asks whether a planar poset with large dimension contains a large standard example. Both problems date from the 1980's, and with the surge of recent results, both problems seem accessible.

Magdalena Tyniec-Motyka

AGH University of Science and Technology

Maximal edge-colorings of graphs

Maximal edge-coloring of a graph G of order n is a proper edge-coloring of a graph G with $\chi'(K_n)$ colors such that no edge of $\overline{G}$ can be attached to G without violating the conditions of proper edge-coloring. We define *the spectrum of maximal edge-coloring* $MEC(n)$ as the set of all m such that there exists a maximal edge-coloring of some graph G, where $|V(G)| = n$ and $|E(G)| = m$. In the talk we present the lower bound for the spectrum. We also show constructions of such colorings. Thus the spectrum is almost completely determined. Moreover, different number of colors used for the coloring will be considered. This is a natural generalization of a problem.

This is joint work with Mariusz Meszka.

Jan Volec

McGill University

Transversal and colorful versions of Mantel's theorem

A classical result of Mantel states that every graph of density larger than 1/2 contains a triangle, and this result is best possible. In this talk, we study two Mantel-inspired problems: The first one asks what is the minimum d such that any triplet of graphs $G1$, $G2$,$G3$ on the same vertex-set each of density larger than d contains a transversal triangle, i.e., three edges uv, vw, wu of $G1$, $G2$, $G3$, respectively. We show that $d = (52-4\sqrt{7})/81$, which is asymptotically best possible witnessed by a construction discovered by Aharoni and DeVos. Moreover, we prove that their construction is asymptotically the only extremal configuration. The second problem, due to DeVos, McDonald and Montejano, states that every k-edge-colored graph where each color class has density more than $1/(2k-1)$ contains a non-monochromatic triangle.

This is joint work with Eric Culver, Bernard Lidický, Florian Pfender and Sergey Norin.

Bartosz Walczak
Jagiellonian University

Sparse Kneser graphs are Hamiltonian

For integers $k \geqslant 1$ and $n \geqslant 2k+1$, the Kneser graph $K(n,k)$ is the graph whose vertices are the k-element subsets of $\{1,\ldots,n\}$ and whose edges connect pairs of subsets that are disjoint. The Kneser graphs of the form $K(2k+1,k)$ are also known as the odd graphs. We settle an old problem due to Meredith, Lloyd, and Biggs from the 1970s, proving that for every $k \geqslant 3$, the odd graph $K(2k+1,k)$ has a Hamilton cycle. The proof is based on a reduction of the Hamiltonicity problem in the odd graph to the problem of finding a spanning tree in a suitably defined hypergraph on Dyck words. As a byproduct, we obtain a new proof of the so-called Middle Levels Conjecture.

This is joint work with Torsten Mütze and Jerri Nummenpalo.

Krzysztof Węsek
Warsaw University of Technology

Lower bounds for coloring of the plane

One of the most challenging questions in discrete geometry is the Hadwiger-Nelson problem: how many colors are needed to color the Euclidean plane so that points at distance 1 receive distinct colors? In other words, what is the chromatic number of the unit distance graph of the plane? For over 60 years, the answer was known to be between 4 and 7. Only recently, Aubrey de Grey made a great breakthrough by improving the lower bound to 5. However, it appears that if we consider a modified problem with a wider distance condition, it is possible to make the gap smaller or even give the exact answer. For $b > 1$, let $G_{[1,b]}$ be the graph with the set of vertices $\mathbb{R}^2$ and adjacency between points at distance in the interval $[1, b]$. We obtain new lower bounds on $\chi(G_{[1,b]})$ for certain values of b. Combined with known upper bounds, this result gives two intervals of values of b for which we exactly determine $\chi(G_{[1,b]})$ to be 7 and 9, respectively. The first interval contains and enlarges the only known set of values of b with determined $\chi(G_{[1,b]})$, coming from the work of Exoo (2005)

This is joint work with Konstanty Junosza-Szaniawski.

Mariusz Zając
Warsaw University of Technology

A short proof of Brooks' theorem

This proof of Brooks' theorem uses only induction and greedy coloring, while avoiding issues of graph connectivity. The argument generalizes easily to some extensions of Brooks' theorem, including its variants for list coloring, signed graphs coloring and correspondence coloring.

Andrzej Żak
AGH University of Science and Technology

The size of a largest component vs. node fault tolerance

We study a problem of minimizing the size of a largest component of a graph by removing a given number of its vertices. For $S \subset V(G)$ let $\Omega(G,S)$ denote the number of vertices in a largest component of $G - S$. Then let

$$\omega(G,k) = \min\{\Omega(G,S) : S \subset V(G), |S| = k\}.$$

This parameter is closely related to the notions of a graph separator and the fragmentability of graphs. Namely, the planar separator theorem by Tarjan and Lipton can be expressed as follows.

Theorem 1 (Planar separator theorem) *Let Π be a planar graph of order n. Then*

$$\omega(\Pi, 4\sqrt{n}) \leq \frac{2n}{3}.$$

On the other hand, a graph G of order n is (a,ϵ)-fragmentable if $\omega(G,\epsilon n) \leq a$. In the talk we establish $\omega(C_n \Box C_n, k)$ for (almost) all values of k. Our main result is the following theorem.

Theorem 2 *Let $k \geq 2n$. Then*

$$\omega(C_n \Box C_n, k) \geq \frac{(n^2-k)^2 + (n^2-k)\sqrt{(n^2-k)^2 - k^2}}{k^2}.$$

The lower bound is best possible. We then apply our result in a problem of node-fault-tolerant graphs introduced by Hayes in 1976 as a graph theoretic model of computer or communication networks operating correctly in the presence of faults.

This is joint work with Jakub Przybyło.

Oscar Zamora Luna
Central European University

Avoiding long Berge cycles

Füredi, Kostochka and Luo generalized a theorem of Erdős and Gallai. The maximum number of edges an r-uniform hypergraph can have without containing a for Berge cycles of length at least k. They determined this number for $k > r + 2$ for $k < r - 1$ and asymptotically for $k = r - 1$. Together with Ergemlidze, Győri, Methuku, Salia and Tompkins the cases $k = r$ and $k = r + 1$ were also solved.